Written Exam for the M.Sc. in Economics winter 2015-16
Advanced Development Economics – Macro aspects
Master's Course
December 21st , 2015
(3-hour closed book exam)

Solution manual

ANSWER A.
This problem is a variant of the model in Ashraf, Q. and O. Galor (2011), Dynamics and stagnation in the Malthusian epoch. *AER* 101: 2003-41, Sections IA–IC, p. 2005-9.

A.1.
Total production is

$$Y_t = (A_t X)^{\alpha} L_t^{1-\alpha}, \qquad \text{where } \alpha \in (0,1), \tag{1}$$

and

$$A_t = A L_t^{\beta}, \qquad \text{where } A > 0 \text{ and } \beta \in (0,1). \tag{2}$$

We get $y_t = \frac{Y_t}{L_t}$ by Inserting (2) into (1):

$$Y_t = (A L_t^{\beta} X)^{\alpha} L_t^{1-\alpha}, \tag{3}$$

and then by dividing (3) by L_t:

$$\begin{aligned} \frac{Y_t}{L_t} &= (A L_t^{\beta} X)^{\alpha} L_t^{-\alpha} = \left(\frac{A L_t^{\beta} X}{L_t} \right)^{\alpha} \\ y_t &= \left(\frac{AX}{L_t^{1-\beta}} \right)^{\alpha} \end{aligned} \tag{4}$$

A.2.
Setup the Lagrangian:

$$\mathcal{L} = n_t^{\gamma} c_t^{1-\gamma} + \lambda (y_t - \rho n_t - c_t),$$

and compute the FOC taking y_t as given:

$$\mathcal{L}_{n_t} = \frac{\partial \mathcal{L}}{\partial n_t} = \gamma n_t^{\gamma-1} c_t^{1-\gamma} - \lambda \rho = 0 \tag{5}$$

$$\mathcal{L}_{c_t} = \frac{\partial \mathcal{L}}{\partial c_t} = (1-\gamma) n_t^{\gamma} c_t^{-\gamma} - \lambda = 0 \tag{6}$$

$$\mathcal{L}_{\lambda} = \frac{\partial \mathcal{L}}{\partial \lambda} = y_t - \rho n_t - c_t = 0. \tag{7}$$

Dividing (5) by (6) we get:

$$\left(\frac{\gamma}{1-\gamma} \right) \frac{c_t}{n_t} = \rho$$

or

$$\left(\frac{\gamma}{1-\gamma}\right)c_t = \rho n_t. \tag{8}$$

Inserting (8) into the budget constraint, we find the optimal level of consumption:

$$\begin{aligned}\left(\frac{\gamma}{1-\gamma}\right)c_t + c_t &= y_t \\ c_t &= (1-\gamma)y_t. \end{aligned} \tag{9}$$

Inserting (9) into the budget constraint, we find the optimal level of children:

$$\begin{aligned}\rho n_t + (1-\gamma)y_t &= y_t \\ n_t &= \left(\frac{\gamma}{\rho}\right)y_t. \end{aligned} \tag{10}$$

Comments on the results: A fraction $(1-\gamma)$ of y_t is allocated to consumption and a fraction γ to child rearing. The positive effect of income on fertility decisions is in accordance with the Malthusian paradigm – or a setup in which an economy is at an early stage of development, and one of the factors is in fixed supply.

A.3.
Total population in $t+1$ is

$$L_{t+1} = n_t L_t. \tag{11}$$

Plugging the optimal level of n_t (10) and the level of y_t (**??**) into (11):

$$\begin{aligned}L_{t+1} &= \frac{\gamma}{\rho}\left(\frac{AX}{L_t^{1-\beta}}\right)^{\alpha} L_t \\ L_{t+1} &= \frac{\gamma}{\rho}(AX)^{\alpha} L_t^{1-\alpha(1-\beta)} \equiv \phi(L_t; A, X, \gamma, \rho, \alpha, \beta)\end{aligned} \tag{12}$$

Check the Inada conditions for ϕ: $\phi(0) = 0$, $\phi_{L_t} > 0$, $\phi_{L_t L_t} < 0$, $\lim_{L_t \to 0}\phi_{L_t} = \infty$, $\lim_{L_t \to \infty}\phi_{L_t} = 0$:

- $\phi(0) = 0$
- $\phi_{L_t} = \frac{\partial L_{t+1}}{\partial L_t} = \frac{\gamma}{\rho}(AX)^{\alpha}\left[1-\alpha(1-\beta)\right] L_t^{-\alpha(1-\beta)} > 0.$
- $\lim_{L_t \to 0}\phi_{L_t} = \infty$, and
- $\lim_{L_t \to \infty}\phi_{L_t} = 0$ since $0 < \alpha(1-\beta) < 1$.
- $\phi_{L_t L_t} = \frac{\partial}{\partial L_t}\left(\frac{\partial L_{t+1}}{\partial L_t}\right) = \frac{\gamma}{\rho}(AX)^{\alpha}\left[1-\alpha(1-\beta)\right]\left\{-\alpha(1-\beta)\right\} L_t^{-\alpha(1-\beta)-1} < 0.$

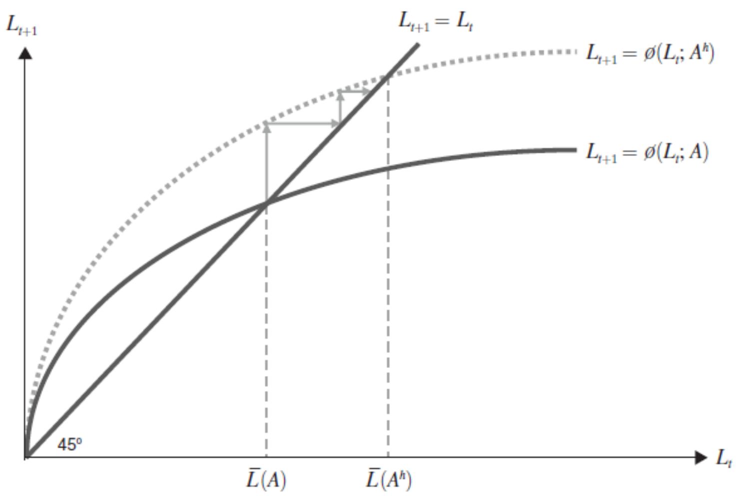
Given that the Inada conditions are fulfilled (or that ϕ starts in the origin it is strictly concave), we can apply the fixed point theorem and conclude the 45 degree line intersects ϕ twice: at 0, and at some level $\overline{L} > 0$.

The trivial steady state $L = 0$ is unstable and will not be an absorbing state for the population dynamics. (It is also ruled out by the condition $L_0 > 0$ in the question,)

$\overline{L}$ is globally stable: wherever L starts (other than 0), it converges to $\overline{L}$. Therefore the system has a unique stable steady state at $\overline{L}$ which can be calculated by setting $L_{t+1} = L_t = \overline{L}$ in the law of motion:

$$\begin{aligned} \overline{L} &= \frac{\gamma}{\rho}(AX)^{\alpha}\left(\overline{L}\right)^{1-\alpha(1-\beta)} \\ \overline{L} &= \left[\frac{\gamma}{\rho}(AX)^{\alpha}\right]^{\frac{1}{\alpha(1-\beta)}} \\ \overline{L} &= \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha(1-\beta)}}(AX)^{\frac{1}{1-\beta}} \end{aligned} \tag{13}$$

The phase diagram looks like:



Source: Ashraf and Galor (2011).

A.4.
Dividing (13) by X:

$$\begin{aligned} \overline{P} &= \frac{\overline{L}}{X} = \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha(1-\beta)}}(AX)^{\frac{1}{1-\beta}}X^{-1} \\ \overline{P} &= \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha(1-\beta)}}X^{\frac{\beta}{1-\beta}}A^{\frac{1}{1-\beta}} \end{aligned}$$

Then

$$\frac{\partial\overline{P}}{\partial A} = \left(\frac{\gamma}{\rho}\right)^{\frac{1}{\alpha(1-\beta)}}X^{\frac{\beta}{1-\beta}}\left(\frac{1}{1-\beta}\right)A^{\frac{\beta}{1-\beta}} > 0.$$

An increase in A generates a transition process in which population gradually increases from its steady-state. The decline in income per capita associated to the increase in population reduces the fertility rate, and allows the system to converge to a new steady-state with a higher level of population density. Therefore, the model predicts that technological progress increases population density.

The empirical evidence supports this prediction. Examining cross national data for the pre-modern period, Ashraf and Galor (2011) confirm a positive impact form technological change on population growth and population density at early stages of development, by exploiting variation in the timing of the Neolithic revolution and an index of technological sophistication as markers for cross-country differences in technological progress during that time.

A.5.
From (4):

$$y_{t+1} = \left(\frac{AX}{L_{t+1}^{1-\beta}}\right)^{\alpha} = (AX)^{\alpha} L_{t+1}^{-\alpha(1-\beta)} \tag{14}$$

Inserting (12) into (14):

$$\begin{aligned} y_{t+1} &= (AX)^{\alpha}\left[\frac{\gamma}{\rho}(AX)^{\alpha} L_t^{1-\alpha(1-\beta)}\right]^{-\alpha(1-\beta)} \\ &= (AX)^{\alpha} L_t^{-\alpha(1-\beta)}\left[\frac{\gamma}{\rho}(AX)^{\alpha} L_t^{-\alpha(1-\beta)}\right]^{-\alpha(1-\beta)} = y_t\left[\left(\frac{\gamma}{\rho}\right) y_t\right]^{-\alpha(1-\beta)} \\ y_{t+1} &= \left(\frac{\rho}{\gamma}\right)^{\alpha(1-\beta)} y_t^{1-\alpha(1-\beta)} \equiv \psi(y_t;\gamma,\rho,\alpha,\beta) \end{aligned} \tag{15}$$

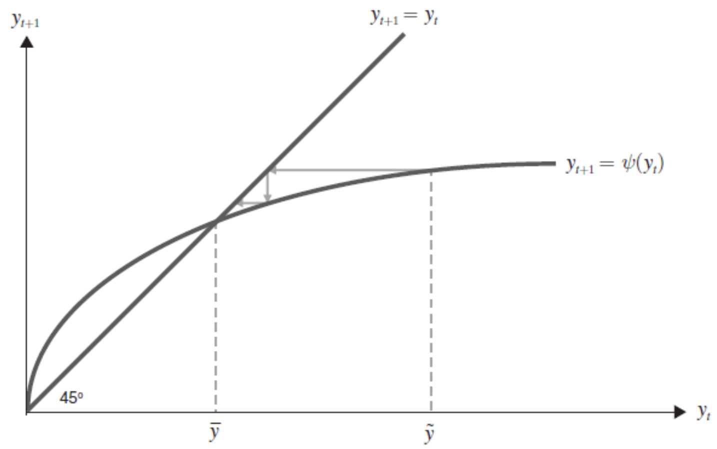
Check the Inada conditions for (15):

- $\psi(0) = 0$.
- $\psi_{y_t} = \frac{\partial y_{t+1}}{\partial y_t} = \left(\frac{\rho}{\gamma}\right)^{\alpha(1-\beta)} [1-\alpha(1-\beta)]\, y_t^{-\alpha(1-\beta)} > 0$.
- $\lim_{y_t \to 0} \psi_{y_t} = \infty$ and,
- $\lim_{y_t \to \infty} \psi_{y_t} = 0$ since $0 < \alpha(1-\beta) < 1$.
- $\psi_{y_t y_t} = \frac{\partial}{\partial y_t}\left(\frac{\partial y_{t+1}}{\partial y_t}\right) = \left(\frac{\rho}{\gamma}\right)^{\alpha(1-\beta)} [1-\alpha(1-\beta)]\,\{-\alpha(1-\beta)\}\, y_t^{-\alpha(1-\beta)-1} < 0$.

The system has a unique stable steady state at $\overline{y}$ which can be calculated by setting $y_{t+1} = y_t$ in the law of motion:

$$\begin{aligned} \overline{y} &= \left(\frac{\rho}{\gamma}\right)^{\alpha(1-\beta)} (\overline{y})^{1-\alpha(1-\beta)} \\ \overline{y} &= \left(\frac{\rho}{\gamma}\right)^{\frac{\alpha(1-\beta)}{\alpha(1-\beta)}} \\ \overline{y} &= \frac{\rho}{\gamma} \end{aligned}$$

The phase diagram looks like:



Source: Ashraf and Galor (2011).

A.6.
While the exogenous shock to technology, A, increases the level of income per worker in the short run, y_t, it does not affect the steady-state level of income per worker, $\overline{y}$:

$$\frac{\partial y_t}{\partial A} = X^{\alpha} L_t^{-\alpha(1-\beta)} \alpha A^{\alpha-1} > 0$$

and

$$\frac{\partial \overline{y}}{\partial A} = 0$$

The empirical literature does support these predictions. Ashraf and Galor (2011) show that during the pre-modern period, higher levels of technological sophistication had insignificant effects on the level of income per capita across countries in a long-run perspective. Taken together with the answer to question A.4., these results imply that during the pre-modern era, increases in technology resulted in larger but not significantly richer populations.

ANSWER B.
Readings:

- Gorodnichenko, Yuriy and Gerard Roland (2015), Culture, Institutions and the Wealth of Nations. Forthcoming in *REStat*.
- Tabellini, Guido (2010), Culture and institutions: economic development in the regions of Europe. *JEEA* 8(4):677-716.

B.1.
Innovation and technological progress is one of the key elements to sustain growth of productivity and economic development in the long-run. As argued by Gorodnichenko and Roland (2015) and Tabellini (2010), cultures that are relatively more individualistc may play a key role in stimulating innovation, because they emphasize personal freedom and achievement, and therefore award social status to personal accomplishments such as important discoveries, innovations, artistic and humanitarian

achievements, or in general actions that make an individual stand out.

Innovation and technological progress can have large and positive dynamic effects on economic growth, and thereby allow an economy to sustain higher levels of economic development in the long-run (as well as other important factors such as higer levels of human capital).

Collectivism, as an alternative dimension to individualism, can also be supportive of higher levels of economic development, since collectivist societies may have an advantage in solving coordination and collective action problems. Given that innovation and faster technological progress will tend to have dynamic effects on economic growth and development, it is possible to hypothesize that the benefits from indivualism are relatively larger than those of collectivism in the long run.

B.2.
Concerns about endogeneity in a regression of y on x might arise due to any reason that creates a correlation between x and the error term. These concerns arise if one suspects of the presence of measurement error, omitted variables in the regression, simultaneity, reverse causality, or some or all of these.

Measurement error, if classical, tends to bias the coefficient of interest towards zero (attenuation bias). A potential solution is relying on other proxies of the explanatory variable. For example, running the same regression by using alternative indices of the main explanatory variable (individualism), would be a way to test whether the results are robust to proxies that arguably have different degrees of measurement error.

To address omitted variables and simultaneity, one typical solution is expanding the set of controls, by chosing variables possibly correlated with the outcome variable, or correlated with both the explanatory and the outcome variables at the same time. For example, the level of human capital could be a factor driving both the level of individualism and income per capita, and one solution could be controlling for predetermined levels of schooling.

Endogeneity due to reverse causality can be addressed for instance by finding a proxy for the variable of interest that is exogenous, or by relying on instrumental variables. In the latter case, the instruments need to be valid (or satisfy an exclusion restriction, and affect the outcome variable only through the explanatory variable), and strong (be highly correlated with the explanatory variable). An example is the use of country's average blood-type distance to the average blood-type in the UK, which according to Gorodnichenlo and Roland (2015) is the relatively more individualistic and homogenous society in their cross-national dataset. The exclusion restriction is satisfied in this case, because blood-type is a neutral genetic characteristic, but transmitted from parents to children in a similar way as features of their culture are transmitted.

ANSWER C.
Readings:

- Acemoglu, D. (2010), Chapter 4: Fundamental Determinants of Differences in

Economic Performance, in "Introduction to Modern Economic Growth," Princeton University Press. Sections 4.1 and 4.3.

- Acemoglu, D., S. Johnson, and J. A. Robinson (2001), The Colonial Origins of Comparative Development: An Empirical Investigation. *AER* 91: 1369-1401. 33 pages.
- Acemoglu, D., S. Johnson, and J. A. Robinson (2002), Reversal of Fortune: Geography and Institutions in the Making of the Modern World Income Distribution. *QJE* 117(4): 1231-1294.
- Acemoglu, D. and J. A. Robinson (2010), The Role of Institutions in Growth and Development. *Review of Economics and Institutions* 1(2): 1-33.

C.1.
Institutions refer to a broad cluster of arrangements that influence economic interactions among economic agents. They can be defined (following Douglass North) as the 'rules of the game', or the set of humanly devised constraints that shape human interaction.

Institutions, which in practice take the form of rules for organization of the society, regulations, laws, and policies, are considered a *fundamental* cause of differences in economic performace because they affect economic incentives, and thereby help to determine levels of investment in innovation, in human and physical capital accumulation, or in other factors that considered *proximate* causes of differences in economic performace.

What distinguishes institutions from culture and geography (as two other fundamental causes of differences in economic development), is that institutions are social choices, or the outcome of collective decisions – which has the corollary that societies can change dysfunctional institutions (or at least have the potential to change them) by embarking on processes of institutional reform.

C.2.
Following AJR, institutions in North Korea can be characterized as *extractive*. Extractive institutions concentrate economic and political power in the hands of a small elite, discourage participation in economic and political life, limit the role of private property and create risk of expropriation, and thereby increase the chances that groups holding political power will extract or capture rents in the economy. Institutions in the South can be characterized as *inclusive*, or *institutions of private property*. Inclusive institutions provide security and protection to a broad cross section of the population, and to offer access to political power, for instance by offering the chance to certain citizens to participate in elections.

Inclusive institutions, as opposed to extractive institutions, provide incentives to undertake investments, and limits to expropriation and capture of rents. Under inclusive institutions, national savings, investments, and accumulation of factors is larger, which facilitates economic growth. Imclusive institutions can also fuel innovation and thereby increase the rate of technological progress, which has the potential to put an economy on a long-term path of sustained growth and development.